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ESTIMATION OF RELIABILITY GROWTH DETERMINATION IN CRACKED SPECIMENS UNDER FATIGUE FAILURE

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Abstract:

A probabilistic analysis of the fatigue crack growth for reliability growth calculation on the mechanical component is presented on the basis of fracture mechanics and theory of random process. The loading is postulated to be stationary, narrow-band random Gaussian process and consequently, randomized Paris-Erdogan law is applicable. As a specific problem, a thin plate having a central crack is analyzed by two analytical methods "stochastic averaging" and "damaged linear accumulation". Simultaneously, the aforementioned plate is being analyzed by the utilization of "NISAII", Software. Analysis made by "NISAII" is on the basis of Monte-Carlo simulation random analysis. In the end, all the results are compared with each other and conclusion is drawn.

Key Words:
Fatigue damaged linear accumulation; Monte Carlo simulation; Random fatigue crack growth; Random loading; Reliability growth; Stochastic averaging

Introduction:

One of the main contributors to the structure failure of an aircraft is a phenomenon known as fatigue. Most of the mechanical components, especially aircraft structure, have been designed on the basis of "Fail safe" and "damage tolerance" concepts. Providing assurance of no failure operation is highly important in these components. Reliability evaluation can provide useful information in the fatigue control.

Evaluation of fatigue effect on mechanical components containing initial crack is rather complex. The complicity arising from crack growth in component that depends on essential properties of material under fatigue load also depends on geometry of component as well as environmental conditions. In the laboratory controlled conditions,

if an experiment is repeated many times over and test conditions remain the same, the results are not deterministic, and in most cases are scattered. Critical elements designed on the basis of the above-mentioned points will be very reliable. One of the important concepts in reliability analysis is the definition of failure criteria, which can estimate probability of useful operation. In mechanical components, existence crack is one of major failure causes, hence reaching crack length to a critical length can be defined as the failure criteria. Equations of fracture analysis are in the form of probability, therefore we can't apply deterministic fracture mechanics. Stochastic behavior should be evaluated by statistical and probability theories for reliability analysis which introduce complicity in analysis results. Reliability can be analyzed with obtained results and proper modeling and solution by stochastic method Thus becomes an important point.
In this regard, there are a host of stochastic methods. Moreover, recently presented methods reduce essential complicity of probability calculations. Stochastic averaging method for fatigue crack measurement in a component under random loading presented by Zhu and Lin (1992)[1] is to name one. B.Kececioglu in 1998 analyzed reliability of mechanical component under wide-band and narrow-band loading by definition of damage function [2]. Results obtained from the aforementioned researches are used as comparative criteria in the presented study here.
In this paper, reliability analysis of a mechanical component is evaluated under critical conditions. In addition, failure cause is assumed to be the fatigue fracture that arises from crack growth. Component behavior is evaluated on the basis of probability fracture mechanics and random vibration theories. Critical conditions are presumed to be simpler assumptions for reducing governing probability complicity. On the basis of these assumptions, the problem is designed and solved by governing theories. Property of a dynamic system is light damping and stationary narrow-band Gaussian random loading process. Consequently, the system response would be come close to the system's resonance response, (i.e. high stress fatigue is taken into account.) "Stochastic averaging method" and "damaged linear accumulation method" is used as analytical solutions. Software analysis (by NISAII) is presented on the basis of Monte-Carlo random simulation method.

Methodology:

In this section, analytical and software methods are used for the reliability growth determination in a cracked mechanical component under fatigue failure. Considering the complicity of methods for reliability growth in this field, it becomes necessary to design a problem with simpler conditions in order to make it possible to expand into general conditions.

Problem Assumption:

Dynamic Analysis

- one-degree of freedom and light damping system
- stationary wide band Gaussian random loading process(white noise)

Fracture Analysis

- only one crack in component
- length and location of crack known

- material behavior, brittle, homogeneous, isotropic
- high stress fatigue

Reliability Analysis

- failure case, fatigue fracture
- failure criteria, a=ac

Model Selection:

Model assumptions should be one-degree of freedom, light damping and wide band loading. Test specimen concerned is thin plate with a central crack, however, it can be expanded to other specimens.

Problem:

Consider a thin square plate 1×1, shown in Fig. 1, with an initial central crack of length $2a_o$ and supporting an infinitely rigid heavy mass *M* at its end. The plate is idealized to be massless, homogeneous, isotropic, and with light damping, and the mass *M* is subjected to a Gaussian load process perpendicular to crack (Y) with a wide-band one-sided spectral density *G*.

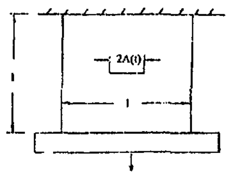

Fig.1 Thin plate containing central crack and mass in its end

Analytical Solution Methods:

For calculation of plate reliability growth presented "stochastic averaging" and "accumulative linear damage" as analytical methods

(1) Stochastic Averaging Method[3]

Model of System Dynamic

For one degree of freedom, the system can be considered as dynamic model of mass, spring and damping system, then the dynamic equation will be:

$$M\ddot{X}(t)+C\dot{X}(t)+g[A(t)]X(t)=\xi(t) \tag{1}$$

The stiffness function g[A(t)] can be approximated by following polynomial expression[4]

$$g(a)=g(0)(1-1.708u^2+3.081u^4-7.036u^6+8.928u^8-4.266u^{10}) \tag{2}$$

Which g(o) is without crack plate stiffness and $g(a_0=0)\cong E.A/L$ and $u=\dfrac{2a}{L}$.

We shall assume that the stress is equal to the displacement multiplied by a constant.

Fracture Analysis

Based on Paris-Erdogan model, the equation for crack growth rate is:

$$\frac{dA}{dt} = \frac{\omega(A)}{2\pi}\alpha(\Delta K)^{\beta} \tag{3}$$

Where:

$$\Delta K = 2\sqrt{\frac{\pi G g(a)}{C}}h(a)R \tag{4}$$

R , stress range, an approximate expression for h(a) has been provided in [4]as following:

$$h(a) = \sqrt{u}(0.467 - 0.514u + 0.960u^2 - 1.421u^3 + 0.782u^4) \tag{5}$$

now eq.(3) can be rewritten as follows:

$$\frac{dA}{dt} = \gamma Q(A)S^{\beta} \tag{6}$$

$$\gamma = \frac{\alpha}{2\pi}(2\sqrt{(\pi G / c)})^{\beta} \tag{7}$$

$$Q(A) = \omega(A)[h(A)\sqrt{(g(A))}]^{\beta} \tag{8}$$

In which S is the stress envelope process and R (t)=2 S(t). Then by solving the above equations, the following transition probability density of crack size A (t) will be yield.

$$q(a,t|a_0,0) = \frac{1}{\sqrt{(2\pi t)}Q(a)\sigma\Phi(m\sqrt{t}|\sigma)}\exp\left\{-\frac{\left[\int_{a0}^{a}\frac{du}{Q(u)} - mt\right]^2}{2\sigma^2 t}\right\}; \quad a \geq a_0 \tag{9}$$

Where:

$$m = \gamma\,\Gamma(1+\beta|2) \tag{10}$$

$$\sigma^2 = \gamma^2\sum_{n=1}^{\infty}c_n^2\tau_n \tag{11}$$

$$c_n = \sum_{k=0}^{n}\frac{(-1)^k n!}{(n-k)!\,(k!)^2}\Gamma(1+k+\beta|2) \tag{12}$$

$$\tau_n = 2\int_0^{\infty}\rho^{2n}(\tau)d\tau \tag{13}$$

$\rho=\exp[-C\,\tau/(2M)]$ correlation coefficient of loading process.
The reliability function follows:

$$R(t,a_{cr}|a_0,0)=\int_{a_0}^{a_{cr}} q(a,t|a_0,0)da = 1-\frac{\Phi\left[\dfrac{mt-z_{cr}}{\sigma\sqrt{t}}\right]}{\Phi\left[m\sqrt{t}|\sigma\right]} \tag{14}$$

Where:

$$z_{cr}=\int_{a_0}^{a_{cr}}\frac{da}{Q(a)}$$

(2) ***The Linear Accumulation Hypothesis in random fatigue crack growth***
Paris-Erdogan model is given in the form [5]

$$\frac{da}{dn}=C(\Delta k)^m \tag{15}$$

Where:

$$\Delta k=\Delta S\sqrt{\pi a}\,\eta(a)$$

When the crack size is very small in comparison with the component dimensions, the above equation can be rewritten as

$$\frac{da}{dn}=CG(a)(\Delta S)^m \tag{16}$$

Where:

$$G(a)=[\eta(a)\sqrt{\pi a}\,]^m$$

and Eq.(16) can be written

$$\Psi(a)=\int_{a_0}^{a}\frac{dz}{G(z)} \tag{17}$$

Where

$$\frac{d\Psi(a)}{dn}=C(\Delta S)^m$$

the damage indicator is defined as:

$$D=\frac{\Psi(a)}{\Psi(a_c)} \tag{18}$$

It can be shown that when a=a0, D=1; and when a=ac, D=1. The increase rate of the damage indicator therefore is

$$\frac{dD}{dt}=\frac{2^m fC(\Delta S)^m}{\Psi(a_c)} \tag{19}$$

Where S is the stress amplitude process, and $S=\frac{\Delta s}{2}$.

Definite the coefficient A as

$$A=\frac{\Psi(a_c)}{2^m c} \text{，} b=m \tag{20}$$

It can be shown in [5] that the damage accumulation under stationary stress loading process is a normally distributed random variable; i.e.,

$$D(t)\propto N[\bar{D}(t),\sigma_{D(t)}] \tag{21}$$

For narrow band stress process:

$$\bar{D}(t)=\frac{\omega_n}{2\pi}\frac{t}{A}(\sqrt{2}\sigma_x)^b\Gamma(\frac{b}{2}+1) \tag{22}$$

$$\sigma_{D(t)}=\frac{(\sqrt{2}\sigma_x)^b}{A}\Gamma(\frac{b}{2}+1)\sqrt{\frac{\omega_n}{2\pi}\frac{\varphi_1(b)}{\zeta}} \tag{23}$$

$\varphi_1(b)$ which is given in[2] and α and β definite:

$$\alpha=\frac{v}{\sqrt{u}} \quad ,\beta=\frac{1}{u} \tag{24}$$

$$u\cong\frac{\omega_n}{2\pi A}(\sqrt{2}\sigma_x)^b\Gamma(\frac{b}{2}+1) \tag{25}$$

$$v\cong\frac{(\sqrt{2}\sigma_x)^b\Gamma(\frac{b}{2}+1)}{A}\sqrt{\frac{\omega_n}{2\pi}\frac{\cdot\varphi_1(b)}{\zeta}} \tag{26}$$

The reliability is then given by

$$R(t)=\Phi\{\frac{1}{\alpha}[(\frac{t}{\beta})^{-1/2}-(\frac{t}{\beta})^{1/2}]\} \tag{27}$$

Analysis:

Reliability-Time curve shown in Fig.2 for analytical method.

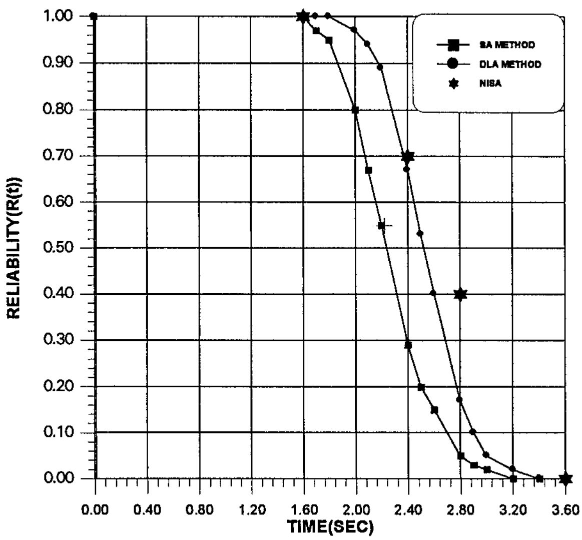

Fig.2 curve of Reliability –Time with two methods

Physical Data of Problem:

$2a_0$=0.00254 [m]
2ac=0.01016 [m]
C=3502.536 [Kg/S]
E=68.95 [GPa]
G=1.243[N^2/HZ]
K_{IC}=91.3 [Mpa $\sqrt{m}$]

L= 0.0254[m]
M= 54.28 [Kg]
Sy=560[Mpa]
th = 0.00254 [m]
α=0.1354E-8
β=2.25

Computer Solution Method:

The first dynamic analysis and former fatigue fractures analysis are carried out on the basis of Monte-Carlo method. Finite element method is used as numerical solution, which is carried out by "NISAII" software. At the beginning, Monte-Carlo analysis method is presented for random solution.

Monte-Carlo Simulation Method:

This method is the simulation of an experiment with a computer. The set of random numbers are generated for random parameters at the beginning, then random numbers are constituted in response equations from which the set of random numbers for identification of random behavior is obtained. This set is analyzed by statistical methods, nonetheless it is possible to obtain quantitative and qualitative responses. Applications of this method are enormous. In this paper flow chart of method is shown in Fig. 3.

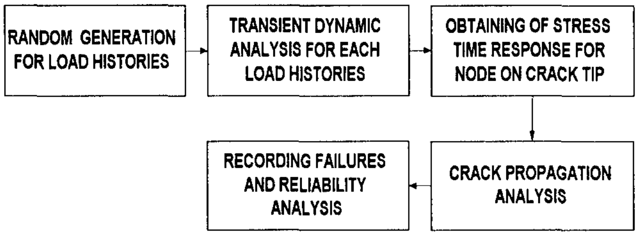

Fig.3 flow chart of reliability analysis by Monte-Carlo method

Dynamic Analysis:

Random vibration analysis is carried out on the basis of given dynamic data and white noise excitation $G_E = 1.24326 \ [\frac{N^2}{HZ}]$ for obtained RMS of response that required in "Damage linear accumulation method" and determination of excitation frequency band that required in "Calculation of excitation standard deviation for transient dynamic analysis". Therefore, stress response S_{yy} is evaluated for node on the crack tip. Power Spectral Density (PSD) of S_{yy} stress response is shown in Fig. 4. A RMS value for loading process (force excitation) and stress dynamic response is obtained.

$$(RMS)_{EX.} = 1.6[N] \qquad\qquad (RMS)_{Res} = 1.5E2[Pa]$$

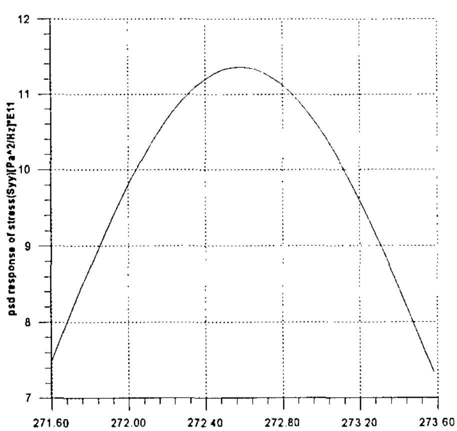

Fig.4 PSD Response of stress S_{yy} in frequency domain related to node on crack tip

Results of Crack Growth and Reliability:

Results of reliability analysis are presented in Tab.I for data of final crack length at time t=1.6 sec, and calculated reliability value is written only for other times.

Tab.I Results of Reliability and Fracture Analysis (a_c = 5.08[mm])

NUMBER OF SIMULATION	1	2	3	4	5	6	7	8	9	10
a_f [mm] at Time=1.6	3.26	3.6	3.92	2.76	4.3	3.64	3.21	3.48	3.7	3.08
FAILURE($a=a_c$)	NO	NO	NO	NO	NO	NO	NO	NO	NO	NO
NO FAILURE	10	7	4	0.	0.					
TIME(S)	t=1.6	t=2.4	t=2.8	t=3.6	t=4					
R(t)	10/10 =1	0.7	.4	0	0					

Conclusion:

Estimated reliability is shown in Fig. 2 by three methods "Stochastic Averaging (SA)", "Damage Linear Accumulation (DLA)" and "Software (NISAII)". Proceeding from Fig. 2 one can see that the results of this method are agreeable and very compatible with two methods mentioned previously. **The new proposed method enables us to analyze wider range of specimens in more complicated conditions**.

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